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MULTI-ENTITY STOCK DEPENDENT MODEL WITH CAPACITY AND MANUFACTURE COST RESTRAINT'S

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Abstract: Inventory has an impact on the manufacturing process as well as supply chain operations. The fundamental goal of this research paper is to optimize the cost associated with inventories and to provide flow less continuous production process in time. Normally, demand rate of any entity in inventory control model are treated as predictable and at the same time constant too, and that the cost associated to unit inventory must be independent and non-variable in nature. Nevertheless, in practical circumstances, the unit price and demand rate of an entity may be interconnected. When the asking for an article is enormous, an entity is manufactured in huge quantities and the static charges of manufacturing being diffused over a multiple component. Henceforth, per unit article cost decreases significantly. i.e., per unit article cot and the demand of an article are related under inverse variation. So, better to consider the demand rate of an article as a variable constraint than to fixed one. In this research article, a mathematical model for multiple articles through permitted and restricted shortage and per article cost based on demand accompanied by upper and lower limits viz restricted storage space and manufacturing expenses has been constructed. Overall, investigating the simultaneous effect of storage space and manufacturing expenses in an inventory model provides valuable insights that enable cost optimization, resource allocation, capacity planning, and risk mitigation. It helps companies make informed decisions and improve their overall operational efficiency and profitability. The Multi-Entity Stock Dependent Model with Capacity and Manufacture Cost Restraints can be customized and used in a variety of sectors that include managing inventory across numerous entities and complicated supply chain networks. Here are a few examples of industries that can benefit from such a model: manufacturing industry, the retail and distributor sector, e-commerce companies, pharmaceutical and healthcare industry, automotive industry and food and beverage industry. The article cost is explored at this juncture in a fuzzy atmosphere and solutions of the model being obtained through KKT condition. Finally, a conclusion is offered in the final portion.

Keywords: *Integrated inventory model, restricted shortages, KKT, fuzzification, variable constraint.*

1. Introduction

The economic order quantity model has an essential and key functioning in the field of inventory. When applying the EOQ model to particular practical scenarios encountered in real life, it is difficult to precisely estimate the cost associated with the various terminology of inventory viz, setup, carrying, shortage, demand etc. Only approximate values may be found. Mostly, the situation which are unpredictable and not certain are studied under the influence of stochastic inventory theory.

Inventory control is a crucial area for both real-world applications and research reasons. The most often used inventory model is the Economic Order Quantity model, in which the sequential operations are classed as supply and demand. The first quantitative treatment of inventory was the basic EOQ model. This model was created by Harris (1915) later, Hadley & Whitin (1963) analysed several inventory methods.Abu Hashan Md Mashudand et al. (2021), Mishra U et al. (21), [1]Rahman et al. (2022), [2]Rahman et al. (2022) studies the two warehouse inventory problems to get rid of stockout situation but renting or owning two warehouse increases the investment and overall profitability of the entire model decreases. Miah et al. (2021) developed limited time price discount inventory model but restricted this model for the electronic products only. This model could be more generalize to cover more industries.[1]Roy D et al. (2022), [2]Roy D et al. (2022) studies the inventory models with preservation technology and cap-and-trade policy. Sultana et al. (2022) described the role of the discount policy and its impact on the inventory control model. They extended the earlier work of Shaikh A. A. et al. (2017, 2020) in which author considered fully backlogged inventory model and application of preservation facility with ramp type demand. Md. Alamin Khan et al. (2017) proposed the solution of nonlinear system of equation that could be raised during the development of mathematical formulation of inventory model.

The fundamental goal of this mathematical model is to optimize the cost associated with inventories and to provide flow less continuous production process in time. Normally, demand rate of any entity in inventory control model are treated as predictable and at the same time constant too, and that the cost associated to unit inventory must be independent and non-variable in nature. Nevertheless, in practical circumstances, the unit price and demand rate of an entity may be interconnected. When the asking for an article is enormous, an entity is manufactured in

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huge quantities and the static charges of manufacturing being diffused over a multiple component. Henceforth, per unit article cost decreases significantly. i.e., per unit article cot and the demand of an article are related under inverse variation. So, better to consider the demand rate of an article as a variable constraint than to fixed one.

In this chapter, a mathematical model for multiple articles through permitted and restricted shortage and per article cost based on demand accompanied by upper and lower limits viz restricted storage space and manufacturing expenses has been constructed. The article cost is explored at this juncture in a fuzzy atmosphere and solutions of the model being obtained through KKT condition. Finally, a conclusion is offered in the final portion.

2. K-K-T Conditions

Taha (2007) presented how to achieve the optimum solution of a nonlinear programming issue associated to inequality restrictions by applying the Kuhn-Tucker criteria. The construction of the Kuhn-Tucker conditions is based on the Lagrangean approach. Assume that the issue is stated by

Minimize $y = f(x)$ ……..1

Associated to $h_\sigma(x) \geq 0, \sigma = \{x: x \in N\}$. ……..2

The non-negative restrictions $x \geq 0$, if any, are included into the limitations of a natural numbers. The inequality restrictions can be imposed to the equations by implementing non-negative slack variables. Suppose s_σ^2 the amount of slack eliminated from the σ^{th} constraint $h_\sigma(x) \geq 0$.

Let $\varepsilon = (\varepsilon_1, \varepsilon_2, \dots\dots, \varepsilon_n)$, ……..4
$h(x) = (g_1(x), g_2(x), \dots, g_n(x))$ ……..5
and $s^2 = (s_1^2, s_2^2, \dots\dots, s_n^2)$ ……..6

Then the Lagrangean functions are given by

$$H(x, s, \varepsilon) = f(x) - \varepsilon[h(x) - s^2] \quad \dots\dots..7$$

Considering the fractional differentiation of H associated to $x, s,$ and ε criteria that are also sufficient if the objective function and solution domain meet the following limitations:

Sense of optimization	Prerequisite conditions	
	Objective function	Solution space
Maximization	Concave	Convex Set
Minimization	Convex	Convex Set

The following is a summary of the requirements for determining if the Kuhn-Tucker conditions are satisfied:

Problem	Kuhn-Tucker conditions	
1. Max $z =$ f(x)associated to $k^\sigma(x) \leq 0, x \geq 0, \sigma = 1, 2, \dots n$	$\frac{\partial}{\partial x_j} f(x) - \sum_{\sigma=1}^{n} \varepsilon_\sigma \frac{\partial}{\partial x_j} k^\sigma(x) = 0$	……..8
	$\varepsilon_\sigma k^\sigma(x) = 0, k^\sigma(x) \leq 0, \sigma = 1,2, \dots. n$	……..9
	$\varepsilon_\sigma \geq 0, \sigma = 1,2, \dots\dots n$	……..10
2. Max $z =$ f(x)associated to $k^\sigma(x) \geq 0, x \geq 0, \sigma = 1, 2, \dots n$	$\frac{\partial}{\partial x_j} f(x) - \sum_{\sigma=1}^{n} \varepsilon_\sigma \frac{\partial}{\partial x_j} k^\sigma(x) = 0$	……..11
	$\varepsilon_\sigma k^\sigma(x) = 0, k^\sigma(x) \geq 0, \sigma = 1,2, \dots. n$	……..12
	$\varepsilon_\sigma \geq 0, \sigma = 1,2, \dots\dots n$	……..13

Karush was one who firstly introduced and developed the K-K-T conditions in 1939.

3. Formulation and Evaluation of The Model

Let the stock volume of σ^{th} entity $(\sigma \in N)$ be R_σ for instance $t = 0$. In specified range $(0, T_\sigma (= t_{1\sigma} + t_{2\sigma}))$, demand being fulfilled by systematic declination of stock level. This technique results in the inventory level being 0 at the time $t_{1\sigma}$ then the range is allowed to experience shortages in the domain $(t_{1\sigma}, T_\sigma)$.

Figure 3.1 interpret a mathematical model with stock backorder. R_σ is the maximum inventory quantity and Q is the order quantity for one period? Also, $t_{1\sigma}$ indicates the time needed for the R_σ entities to be required. The length of time during one period over whichbackorder will be incurred will be given byFigure-1.

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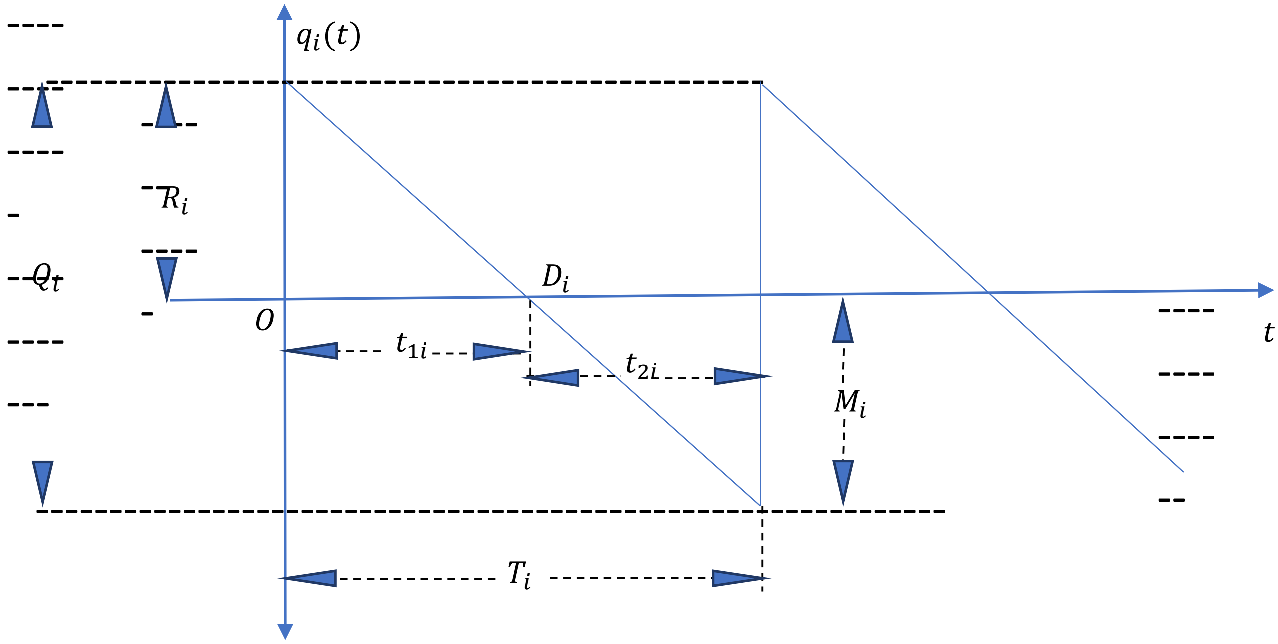

Figure -1 Stock level of the σ^{th} entity

Let $q_\sigma(t)$ be the on-hand inventory at time t $(0 \leq t \leq T)$. In this model, uniform replenishment rate starts with inventory level q_σ. The inventory level decreases with demand. Ultimately the inventory reaches 0 at the end of the cycle time $t_{1\sigma}$.

The derivative function describing inventory level $q_\sigma(t)$ of σ^{th} entity in the range $0 \leq t \leq T_\sigma$ is given by

$$\frac{dq_\sigma(t)}{dt} = \begin{cases} -D_\sigma, \text{for } 0 \leq t \leq t_{1\sigma} \\ -D_\sigma, \text{for } t_{1\sigma} \leq t \leq T_\sigma \end{cases} \quad \dots\dots 14$$

With the conditions $q_\sigma(0) = R_\sigma(= Q_\sigma - M_\sigma), q_\sigma(T_\sigma) = -M_\sigma$ and $q_\sigma(t_{1\sigma}) = 0$ ……15

On every interval a certain quantity of shortfall is permitted and there is a penalty cost m_1 per entity of unmet demand per unit time.

$$\text{For} 0 \leq t \leq t_{1\sigma} \quad \dots\dots 16$$

$$\int_0^t dq_\sigma(t) = -\int_0^t D_\sigma \, dt \quad \dots\dots 17$$

$$q_\sigma(t) - q_\sigma(0) = -D_\sigma t \quad \dots\dots 18$$

$$\text{Hence, } q_\sigma(t) = R_\sigma - D_\sigma(t) \quad \dots\dots 19$$

$$\text{For } t_{1\sigma} \leq t \leq T_\sigma \quad \dots\dots 20$$

$$\int_{t_{1\sigma}}^t dq_\sigma(t) = -\int_{t_{1\sigma}}^t D_\sigma \, dt \quad \dots\dots 21$$

$$q_\sigma(t) - q_\sigma(t_{1\sigma}) = -D_\sigma(t - t_{1\sigma}) \quad \dots\dots 22$$

$$\text{Hence, } q_\sigma(t) = D_\sigma(t_{1\sigma} - t) \quad \dots\dots 23$$

$$\text{Thus } q_\sigma(t) = R_\sigma - D_\sigma(t), \text{for } 0 \leq t \leq t_{1\sigma} \quad \dots\dots 24$$

$$D_\sigma(t_{1\sigma} - t) \text{ for } t_{1\sigma} \leq t \leq T_\sigma \quad \dots\dots 25$$

$$\text{Also } D_\sigma t_{1\sigma} = R_\sigma \quad \dots\dots 26$$

$$M_\sigma = D_\sigma t_{2\sigma} \quad \dots\dots 27$$

$$Q_\sigma = D_\sigma T_\sigma \quad \dots\dots 28$$

The holding cost is related to the cost of carrying (or holding) inventory. This cost frequently encompasses the expenditures such as rent for space, usage for storage, interest on the money locked-up, insurance of stored equipment, manufacturing, taxes, depreciation of equipment and furnishings utilised, etc.

This is derived by evaluating the integral in the range $(0, t_{1\sigma})$ ……29

$$\text{Holding cost} = H_\sigma \int_0^{t_{1\sigma}} q_\sigma(t)dt \quad \dots\dots 30$$

$$= \frac{H_\sigma(Q_\sigma - M_\sigma)^2}{2D_\sigma} \quad \dots\dots 31$$

$$= \frac{H_\sigma T_\sigma(Q_\sigma - M_\sigma)^2}{2Q_\sigma} \quad \dots\dots 32$$

$$\text{Since, } Q_\sigma = D_\sigma T_\sigma \quad \dots\dots 33$$

The penalty cost for running out of stock (i.e., when an entity cannot be provided on the customer's demand) is known as the shortfall cost. This cost encompasses the loss of projected profit via sales of items and loss of goodwill in terms of permanent loss of customers, and it is tied to lost earnings in future sales. Thus, the shortfall cost is calculated by evaluating the integral in the range $(t_{1\sigma}, T_\sigma)$. This is because the shortage develops only after all the present stockpiles are consumed. Hence,

$$\text{Shortage cost} = n_\sigma \int_{t_{1\sigma}}^{T_\sigma} -q_\sigma(t) \, dt \quad \dots\dots 34$$

$$= -n_\sigma \left[\{T_\sigma q_\sigma(T_\sigma) - t_{1\sigma} q_\sigma(t_{1\sigma})\} - \int_{t_{1\sigma}}^{T_\sigma} t(-D_\sigma) \, dt \right] \quad \dots\dots 35$$

$$\text{Since, } dq_\sigma(t) = -D_\sigma \, dt \text{ for } t_{1\sigma} \leq t \leq T_\sigma \quad \dots\dots 36$$

The shortage cost becomes,

$$\text{Shortage cost} = n_\sigma \left[\{T_\sigma(-M_\sigma) - 0\} + D_\sigma \int_{t_{1\sigma}}^{T_\sigma} t \, dt \right] \quad \dots\dots 37$$

$$= n_\sigma T_\sigma M_\sigma - \frac{n_\sigma D_\sigma}{2}[T_\sigma^2 - t_{1\sigma}^2] \quad \dots\dots 38$$

$$\text{But } T_\sigma = \frac{Q_\sigma}{D_\sigma}$$

$$t_{1\sigma} = \frac{R_\sigma}{D_\sigma} = \frac{(Q_\sigma - M_\sigma)}{D_\sigma} \text{ and } Q_\sigma = R_\sigma + M_\sigma \quad \dots\dots 39$$

$$= n_\sigma M_\sigma T_\sigma = \frac{n_\sigma D_\sigma}{2}\left[\frac{Q_\sigma^2}{D_\sigma^2} - \frac{R_\sigma^2}{D_\sigma^2}\right] \quad \dots\dots 40$$

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$$= n_\sigma M_\sigma T_\sigma - \frac{n_\sigma}{2D_\sigma}[\{R_\sigma + M_\sigma\}^2 - R_\sigma^2] \quad \dots\dots 41$$

$$= \frac{n_\sigma M_\sigma^2}{2D_\sigma} \quad \dots\dots 42$$

$$= \frac{n_\sigma M_\sigma^2 T_\sigma}{2Q_\sigma} \quad \dots\dots 43$$

Since, $D_\sigma = \frac{Q_\sigma}{T_\sigma}$

The cost of manufacturing each component of the system is provided by

Production cost $= P_\sigma Q_\sigma$

Additionally, each entity's setup price is provided by

Setup cost $= S_\sigma$

The total cost is defined as the sum of production cost, setup cost, holding cost and shortage cost.

$$TC = p_\sigma Q_\sigma + S_\sigma + \frac{H_\sigma T_\sigma (Q_\sigma - M_\sigma)^2}{2Q_\sigma} + \frac{n_\sigma M_\sigma^2 T_\sigma}{2Q_\sigma}, \text{ for } \sigma = 1, 2, \dots, n. \quad \dots\dots 44$$

The overall average cost of the σ^{th} entity of the system is then given by

$$TC\ (p_\sigma, Q_\sigma, M_\sigma) = p_\sigma D_\sigma + \frac{S_\sigma D_\sigma}{Q_\sigma} + \frac{H_\sigma (Q_\sigma - M_\sigma)^2}{2Q_\sigma} + \frac{n_\sigma M_\sigma^2}{2Q_\sigma} \quad \dots\dots 45$$

The unit price of an entity is often thought of as being constant and independent in nature, and the classical inventory concerns are formed by taking these assumptions into account. By making the assumption that the demand rate and unit price are constants and independent of one another, Silver & Peterson (1985) developed an inventory model. However, in real-world situations, a company's unit price and demand rate may be connected. When there is a high demand for something, it is produced in vast quantities and the fixed production costs are spread among a lot of different goods. As a result, the entity's unit cost declines, making its unit price inversely linked to its demand. This strategy was used by Jung & Klein (2001) to propose and resolve the Economic Order Quantity problem. The inventory model with demand-dependent unit pricing is solved in the current study by utilising the Karush Kuhn-Tucker method.

4. Assumptions of the Inventory Model

Under the following presumptions, a multi-entity stock non-restricted shortage model has been developed.

Instant replenishment is available.

No lead time exists.

Demand is correlated with unit price as $p_\sigma = A_\sigma^\beta D_\sigma^{-\beta}$

where $A_\sigma\ (> 0)$ and $\beta\ (\beta > 1)$ being non-variable, non-imaginary numbers chosen to provide the best fit of the estimated price function $A_\sigma > 0$ is a necessary constraint as individually D_σ and P_σ need to positive.

5. Objective Function of the Model

The objectives of the issue are as follows:

According to the objective of modelling, the total cost of materials must minimise in the system which comprises three parts with shortage cost.

The annual total cost rendering to the fundamental valuation of the mathematical model for economic order quantity is:

Overall cost = manufacture cost + setup cost + holding cost + shortage cost

$$TC\ (p_\sigma, Q_\sigma, M_\sigma) = p_\sigma D_\sigma + \frac{S_\sigma D_\sigma}{Q_\sigma} + \frac{H_\sigma (Q_\sigma - M_\sigma)^2}{2Q_\sigma} + \frac{n_\sigma M_\sigma^2}{2Q_\sigma} \quad \dots\dots 46$$

Substituting for p_σ gives

$$TC\ (D_\sigma, Q_\sigma, M_\sigma) = A_\sigma^\beta D_\sigma^{1-\beta} + \frac{S_\sigma D_\sigma}{Q_\sigma} + \frac{H_\sigma (Q_\sigma - M_\sigma)^2}{2Q_\sigma} + \frac{n_\sigma M_\sigma^2}{2Q_\sigma} \quad \dots\dots 47$$

for $\sigma = 1,2,3,\dots\dots n$

The inventory model's primary goal is to reduce the overall cost specified by

$$\text{Min } TC\ (D_\sigma, Q_\sigma, M_\sigma) = \sum_{\sigma=1}^{n} \left[A_\sigma^\beta D_\sigma^{1-\beta} + \frac{S_\sigma D_\sigma}{Q_\sigma} + \frac{H_\sigma (Q_\sigma - M_\sigma)^2}{2Q_\sigma} + \frac{n_\sigma M_\sigma^2}{2Q_\sigma} \right] \quad \dots\dots 48$$

6. Constraints of the Model

The following limitations have been put on the proposed model: In order to achieve the ideal overall cost, it is necessary to take into account a variety of resource limitations. The amount of warehouse floor space that can be used to store the items is limited.

$$\text{i. e; } \sum_{\sigma=1}^{n} w_\sigma\, Q_\sigma \le W \quad \dots\dots 49$$

The amount that can be invested in overall production costs is finite and may have a maximum investment level.

$$\text{i. e; } \sum_{\sigma=1}^{n} p_\sigma\, Q_\sigma \le B \quad \dots\dots 50$$

$$\Rightarrow \text{i. e; } \sum_{\sigma=1}^{n} A_\sigma^\beta\, D_\sigma^{-\beta} Q_\sigma \le B \quad \dots\dots 51$$

7. Uncertain Stock Model

When $p_\sigma{}'s$ are uncertain decision constraints, the above crisp formulation underneath uncertain situation reduces to

$$\text{Min } TC\ (p_\sigma, D_\sigma, Q_\sigma, M_\sigma) = \sum_{\sigma=1}^{n} \left[A_\sigma^\beta D_\sigma^{1-\beta} + \frac{S_\sigma D_\sigma}{Q_\sigma} + \frac{H_\sigma (Q_\sigma - M_\sigma)^2}{2Q_\sigma} + \frac{n_\sigma M_\sigma^2}{2Q_\sigma} \right] \quad \dots\dots 52$$

associated to the limitations

$$\sum_{\sigma=1}^{n} w_\sigma Q_\sigma \le W \quad \dots\dots 53$$

$$\sum_{\sigma=1}^{n} A_\sigma^\beta D_\sigma^{-\beta} Q_\sigma \le B \quad \dots\dots 54$$

Where $\widetilde{p_\sigma} = A_\sigma^\beta D_\sigma^{-\beta}$ and $\widetilde{p_\sigma}$ represents fuzzification of the parameters.

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8. Karush Kuhn-Tucker Conditions f2or Solving the Stock Model

The impartial function of a stock model is

$$\mathrm{M\sigma n\ TC}\ (D_\sigma, Q_\sigma, M_\sigma) = \sum_{\sigma=1}^{n}\left[A_\sigma^{\beta} D_\sigma^{1-\beta} + \frac{S_\sigma D_\sigma}{Q_\sigma} + \frac{H_\sigma (Q_\sigma - M_\sigma)^2}{2Q_\sigma} + \frac{n_\sigma M_\sigma^2}{2Q_\sigma}\right] \quad \ldots\ldots.55$$

associated to the constraints

$$\sum_{\sigma=1}^{n} w_\sigma Q_\sigma \le W \quad \ldots\ldots.56$$

$$\sum_{\sigma=1}^{n} A_\sigma^{\beta} D_\sigma^{-\beta} Q_\sigma \le B \quad \ldots\ldots.57$$

Here the decision constraints are the demand D_σ, lot size Q and the shortage levelM_1. The problem is, to solve the above inventory model with these decision variables associated to the inequality constraints (3.3) and (3.4) in order to minimize the overall cost function. The problem is solved for a unit entity. For a single entity the objective function and the constraints can be written as follows.

$$\mathrm{Min\ TC}\ (D, Q, M) = \sum_{\sigma=1}^{n}\left[A^{\beta} D^{1-\beta} + \frac{SD}{Q} + \frac{H(Q-M)^2}{2Q} + \frac{nM^2}{2Q}\right] \quad \ldots\ldots.58$$

associated to the inequality constraints

$$wQ \le W \quad \ldots\ldots.59$$

$$A^{\beta} D^{-\beta} Q \le B \quad \ldots\ldots.60$$

To minimize the objective function, the Lagrangean function has been constructed by introducing the variables $s_1$and s_2 as follows:

9. Relationship Function

The relationship function for the vague variable P_σ is defined as follows

$$\mu_{p_\sigma}(X) = \begin{cases} 1, p_\sigma \le L_{L_\sigma} \\ \dfrac{U_{L_\sigma} - p_\sigma}{U_{L_\sigma} - L_{L_\sigma}}, L_{L_\sigma} \le p_\sigma \le U_{L_\sigma} \\ 0, p_\sigma \ge U_{L_\sigma} \end{cases}$$

Here U_L and L_L are superior bound and inferior bound of P_σ correspondingly.

10. Numerical Example

To illustrate the suggested mathematical model for stock with and without shortage instances, the following input data are examined in correct units for a single entity. A numerical example has been constructed for a single entity with the set of input parametric values given in Table-I.

Table-1 The input values of parameters in the mathematical model

Parameter	Notation	Value (in rupees)
Number of entities	n	1
Constant	A_1	20
Setup cost of the entity 1	S_1	80
Holding cost of the entity 1	H_1	0.7
Storage space for the entity 1	w_1	3sq.ft.
Storage space available	W	280sq.ft.
Total investment cost	B	40
Shortage cost per unit entity	n_1	10
Lower limit of the component cost of the entity 1	L_{L_1}	1
Upper limit of the component cost of the entity 1	U_{L_1}	2

For the above data, the objective function becomes

$$G = 20^{\beta} D^{1-\beta} + 80DQ^{-1} + 0.35(Q-M)^2 Q^{-1} + 5M^2 Q^{-1} - \varepsilon_1(280 - 3Q - s_1^2) - \varepsilon_2(40 - 20^{\beta} D^{-\beta} Q - s_2^2) \quad \ldots\ldots.61 \quad \ldots\ldots.62$$

$$G = 20^{\beta} D^{1-\beta} + 80DQ^{-1} + 0.35Q - 0.7M + 5.35M^2 Q^{-1} - \varepsilon_1(280 - 3Q - s_1^2) - \varepsilon_2(40 - 20^{\beta} D^{-\beta} Q - s_2^2) \quad \ldots\ldots.63$$

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Differentiating (63) partially with respect to D, Q and M respectively we get

By the Kuhn-Tucker conditions

$$\frac{\partial G}{\partial D} = 0 \Rightarrow (1-\beta)20^{\beta}D^{-\beta} + 80Q^{-1} - \beta\varepsilon_2 20^{\beta}D^{-\beta-1}Q = 0 \quad \ldots\ldots.67$$

$$\frac{\partial G}{\partial Q} = 0 \Rightarrow 80DQ^{-2} + 0.35 - 5.35\,Q^{-2}M^2 + 3\varepsilon_1 + \varepsilon_2 20^{\beta}D^{-\beta} = 0 \quad \ldots\ldots.68$$

$$\frac{\partial G}{\partial M} = -0.7 + 10.7Q^{-1}M = 0 \quad \ldots\ldots.69$$

Hence, an optimal solution has been obtained by solving the Equations (6), (7) & (8) by implementing K-K-T conditions with demand D, portion size Q and the deficiency level M as the decision constraints by varying the parametric value β. Also, an optimum solution has been obtained by fuzzifying the unit cost and the results are discussed in Table 2.

The value of the parameter β has been chosen between 2 and 3. The most suitable values are obtained for the parametric values β such as 2.4, 2.5, 2.6 and 2.8 by trial-and-error method that minimizes the objective function.
A sensitivity analysis for optimum solution with shortages corresponding to the parameter β is given in Table 2.

Table-2 Optimal results for the model with shortages

β	P_1	μ_{p_σ} value	D_1	Q_1	M_1	Expected Total cost
2.4	1.2853	0.7147	18.014	62.092	4.061	55.151
2.5	1.3053	0.6947	17.978	62.899	4.114	54.872
2.6	1.3278	0.6722	17.934	63.627	4.161	54.633
2.8	1.3806	0.6194	17.824	64.834	4.240	54.197

In Table 2, a study of expected total cost with demand and lot size including shortages is given for different values of β. We can conclude that when demand decreases, lot size increases but the annual total cost decreases.

$$\frac{\partial G}{\partial D} = (1-\beta)20^{\beta}D^{-\beta} + 80Q^{-1} - \beta\varepsilon_2 20^{\beta}D^{-\beta-1}Q \quad \ldots\ldots.64$$

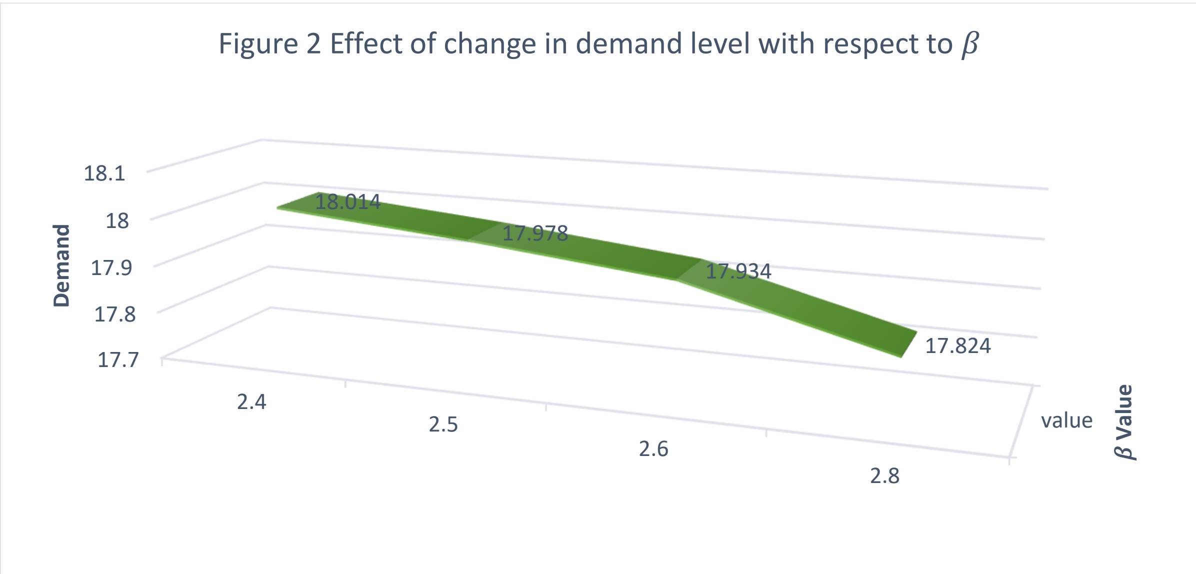
$$\frac{\partial G}{\partial Q} = 80DQ^{-2} + 0.35 - 5.35\,Q^{-2}M^2 + 3\varepsilon_1 + \varepsilon_2 20^{\beta}D^{-\beta} \quad \ldots\ldots.65$$

$$\frac{\partial G}{\partial M} = -0.7 + 10.7Q^{-1}M \quad \ldots\ldots.66$$

From the above table it follows that 1.2853 has the extreme relationship value 0.7147. Henceforth the enforced optimal resolution is $Q_1 = 62.092$, $D_1 = 18.014$, $M_1 = 4.061$ and Minimum expected total cost 55.151.

11. Sensitivity Analysis

Sensitivity examination is investigated to check in what way the productivity of the mathematical model is influenced by modifications or errors in its input parameters based on the numerical example. The outcomes are demonstrated through the assistance of numerical example. A model with and without shortage is discussed in this chapter.
One of the most fundamental inventory models is the stochastic model. The model is significant because it continues to be one of the most widely used inventory models in the sector and acts as a foundation for more sophisticated inventory models.
The values of ambiguous variables needed for decision-making are obtained by solving the inventory model using the Kuhn-Tucker condition technique. The maximum membership function value of 0.7147 corresponds to the best values of the choice variables and the overall cost. Hence the optimal solution is $Q_1 = 62.092$, $D_1 = 18.014$, $M_1 = 4.061$ and $TC = 55.151$.Results due to different values of β for the model has been calculated and depicted in the following Figures 2, 3, 4, 5 and 6.



The above Figure 2 shows that as the value of the parameter β increases from 2.4 to 2.8, the value of the demand decreases from 18.014 to 17.824.

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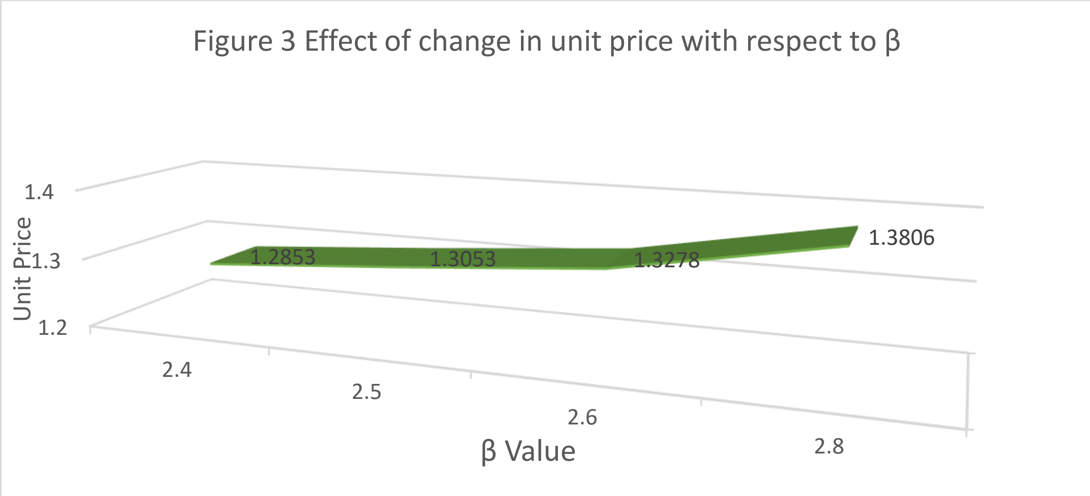
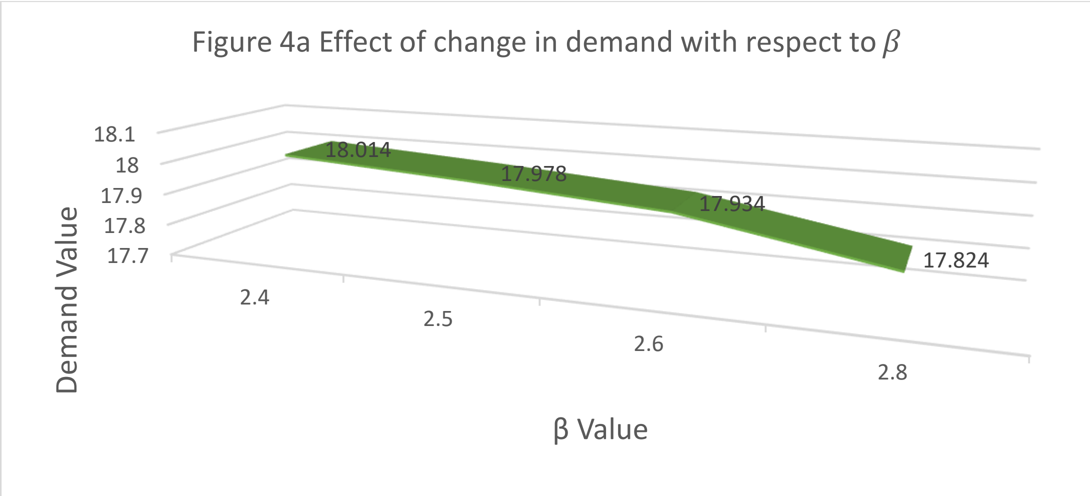
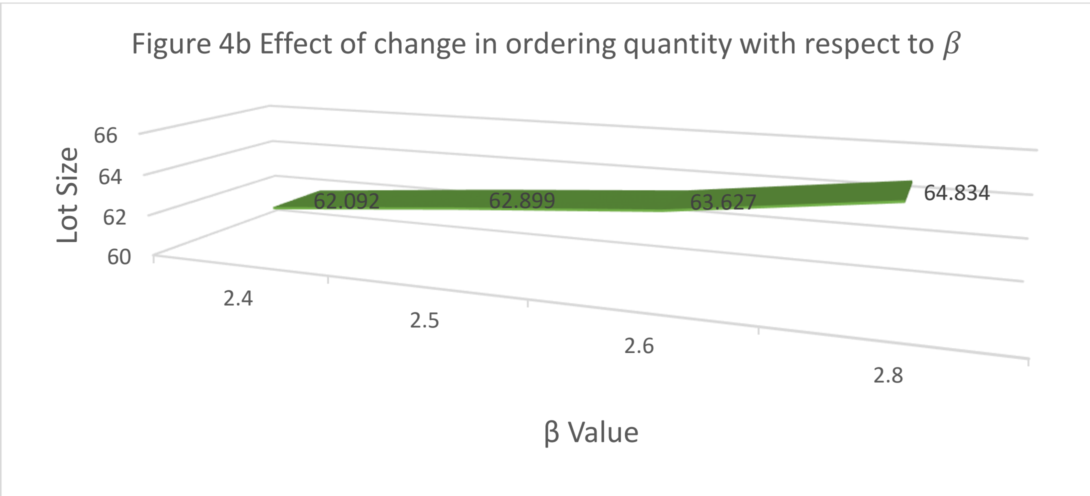


The above Figure 3 shows that as the value of the parameter β increases from 2.4 to 2.8, the value of the unit price increases from 1.2853 to 0.1.3806.





The above Figure 4 shows that as the value of the parameter β increases from 2.4 to 2.8, the value of the lot size increases from 62.092 to 64.834.

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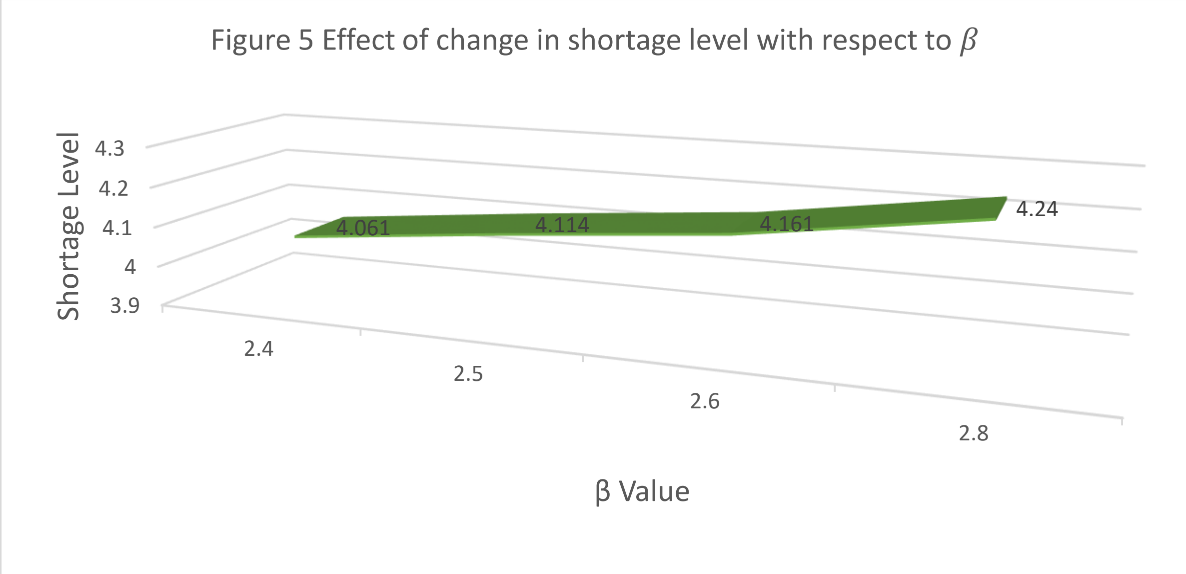
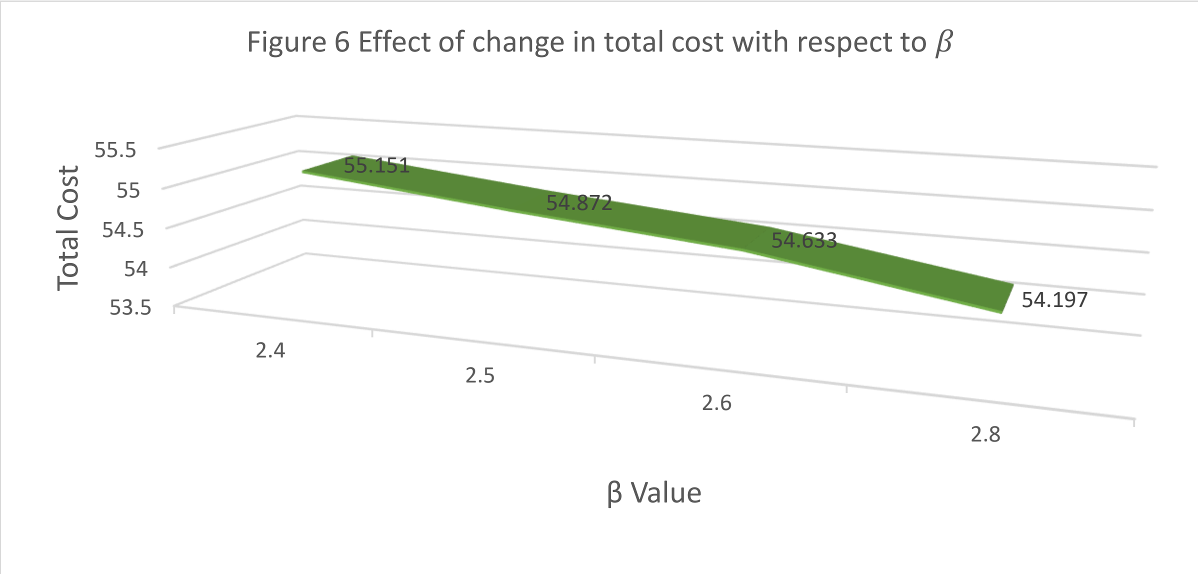


The above Figure 5 shows that as the value of the parameter β increases from 2.4 to 2.8, the value of the shortage level increases from 4.061 to 4.240.



The above Figure 6 shows that as the value of the parameter β increases from 2.4 to 2.8, the value of the annual total cost decreases from 55.151 to 54.197.

12. Inventory Model Without Shortages as a Singular Instance

It is possible to get the scenario without a shortage by inserting in the preceding model. The only variables that affect the total cost function in this scenario are demand and lot size. An inventory model without shortages can be reduced to

$$M\sigma n\ TC\ (D_\sigma, Q_\sigma) = \sum_{\sigma=1}^{n}\left[A_\sigma^\beta D_\sigma^{1-\beta} + \frac{S_\sigma D_\sigma}{Q_\sigma} + \frac{H_\sigma Q_\sigma}{2}\right] \quad \ldots\ldots 70$$

associated to the constraints

$$\sum_{\sigma=1}^{n} w_\sigma Q_\sigma \le W \quad \ldots\ldots 71$$

$$\sum_{\sigma=1}^{n} A_\sigma^\beta D_\sigma^{-\beta} Q_\sigma \le B \quad \ldots\ldots 72$$

For a single entity the inventory model can be stated as

$$M\sigma_n TC\ (D, Q) = A^\beta D^{1-\beta} + \frac{SD}{Q} + \frac{HQ}{2} \quad \ldots\ldots 73$$

associated to the constraints

$wQ \le W$ and $\quad \ldots\ldots 74$

$$A^\beta D^{-\beta} Q \le B \quad \ldots\ldots 75$$

The Lagrangean function corresponding to this objective function can be written as

$$G = A^\beta D^{1-\beta} + SDQ^{-1} + 0.5HQ - \varepsilon_1(W - wQ - s_1^2) - \varepsilon_2(B - A^\beta D^{-\beta} Q - s_2^2) \quad \ldots\ldots 76$$

Differentiating the above functionpartially with respect to D and Q gives the following derivatives.

$$\frac{\partial G}{\partial D} - (1-\beta)A^\beta D^{-\beta} + SQ^{-1} - \varepsilon_2 \beta A^\beta D^{-\beta-1} Q \quad \ldots\ldots 77$$

By Karush Kuhn-Tucker conditions

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$$\frac{\partial G}{\partial D} = 0 \Longrightarrow (1-\beta)A^{\beta}D^{-\beta} + SQ^{-1} - \varepsilon_2\beta A^{\beta}D^{-\beta-1}Q = 0 \quad \ldots\ldots 78$$

$$\frac{\partial G}{\partial D} = 0 \Longrightarrow -SDQ^{-2} + 0.5H + \varepsilon_1 w + \varepsilon_2 A^{\beta}D^{-\beta} = 0 \quad \ldots\ldots 79$$

Solving the Equations (14) and (15) gives the required optimum solution. Ideal resolution in deprived of shortages is assumed in the subsequent

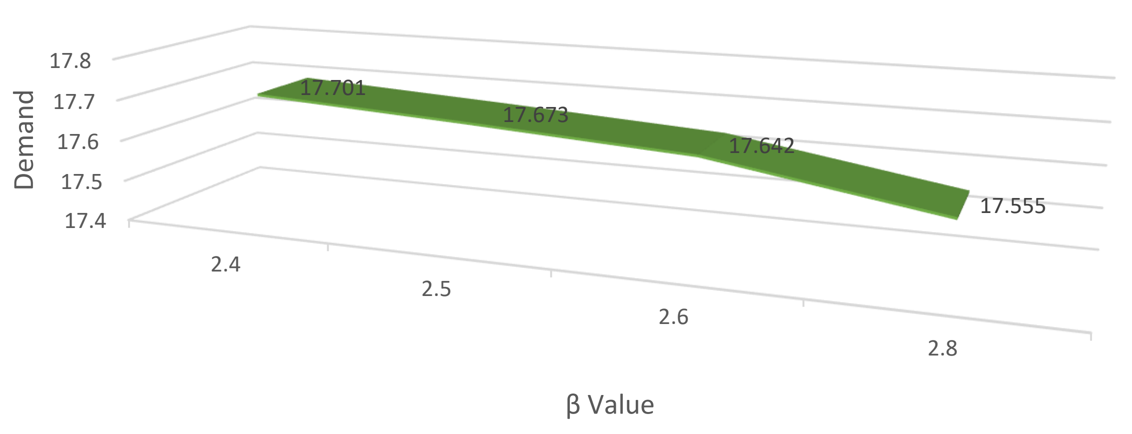
Table 3 for the same set of input values given in Table I.

Table 3 Optimal values for various values of for no-shortage case

β	p_1	μ_{p_1} value	D_1	Q_1	Expected Total cost
2.4	1.3405	0.6595	17.701	59.538	56.519
2.5	1.3624	0.6376	17.673	60.268	56.282
2.6	1.3857	0.6143	17.642	60.967	56.059
2.8	1.4407	0.5593	17.555	62.223	55.649

In this case, it follows that the minimum annual total cost corresponds to the determined relationship function value is 0.6595. Hence the optimal result satisfying the constraints are $D_1 = 17.701$, $Q_1 = 59.538$ and the minimum total cost is equal to 56.519. The graphical representations are shown in the Figure 7, 8,9 and 10.

Figure 7 Effect of change in demand level with respect to β



The above Figure 7 shows that as the value of the parameter β increases from 2.4 to 2.8, the value of the demand decreases from 17.701 to 17.555.

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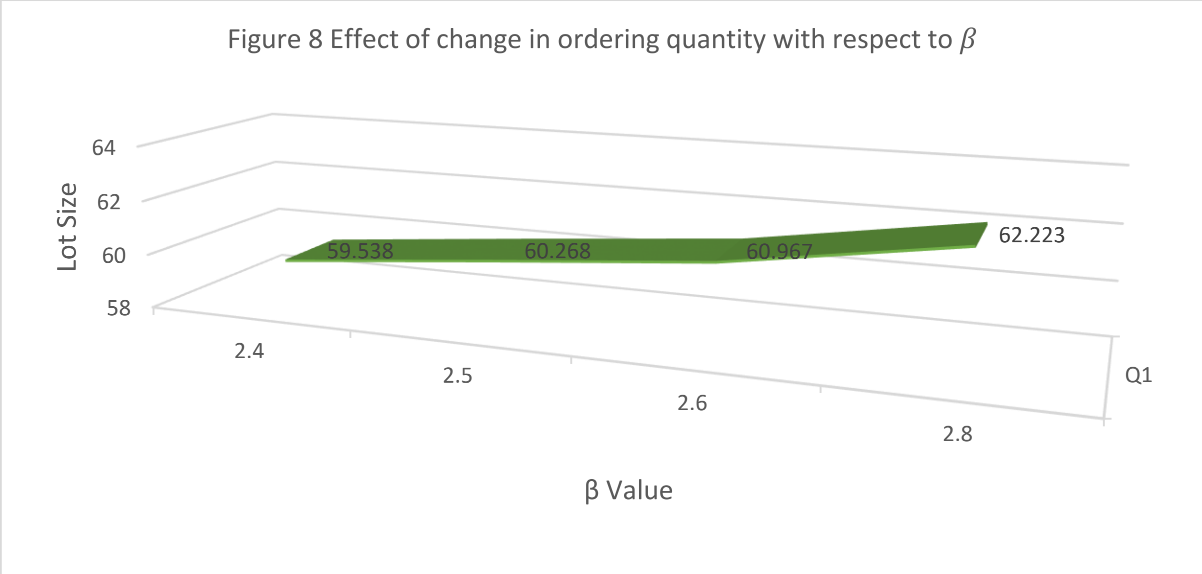
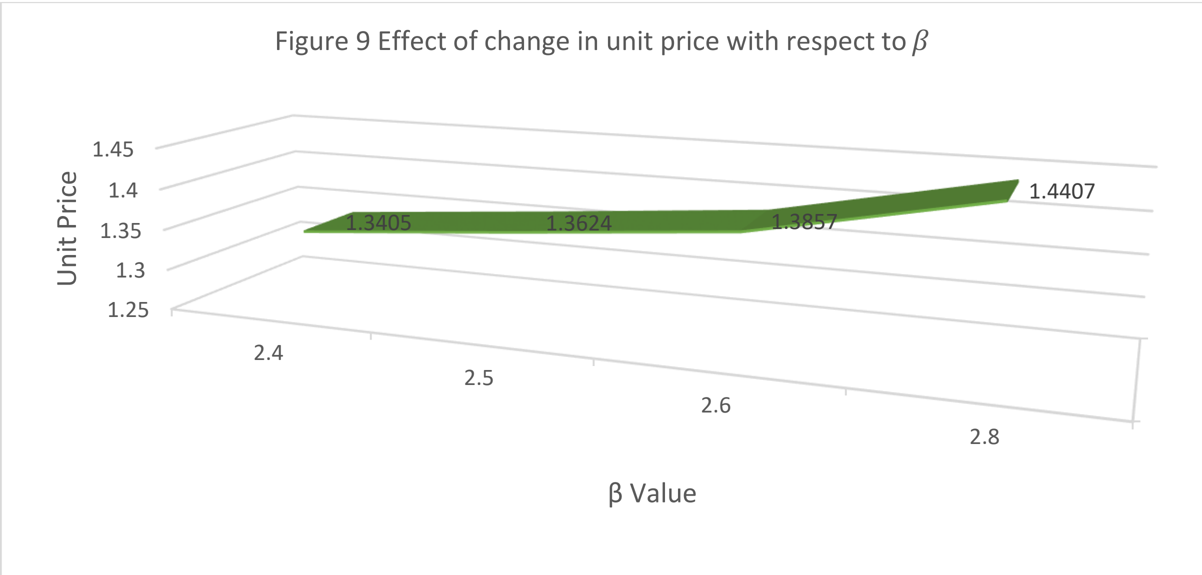


The above Figure 8 shows that as the value of the parameter β increases from 2.4 to 2.8, the value of the lot size increases from 59.538 to 62.223.



The above Figure 9 shows that as the value of the parameter β increases from 2.4 to 2.8, the value of the unit price increases from 1.3405 to 1.4407.

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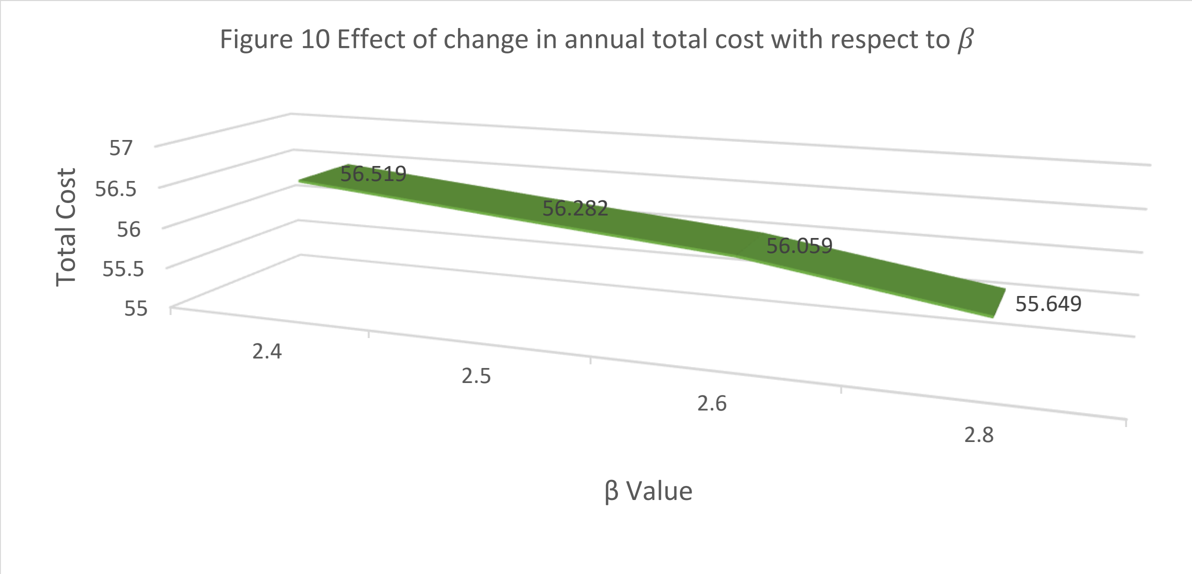

Figure 10 Effect of change in annual total cost with respect to β

The above Figure 10 shows that as the value of the parameter β increases from 2.4 to 2.8, the value of the total cost decreases from 56.519 to 55.649.

13. Summary

In this research article, a mathematical model for multiple articles through permitted and restricted shortage and per article cost based on demand accompanied by upper and lower limits viz restricted storage space and manufacturing expenses has been constructed. The article cost is explored at this juncture in a fuzzy atmosphere and solutions of the model being obtained through KKT condition. A comparison study of the findings for with shortage case(Table-2) and without shortage case(Table-3) is done. In the numerical examples(Table-1), it is found that the optimum total cost in the shortage scenario is less than that of the without shortage case.

The purpose behind developing this model is to provide several managerial insights and constructive conclusion such as: the model aids managers in deciding how best to distribute resources among various supply chain organizations.To maximise overall performance, managers may take well-informed decisions on production schedules, capacity utilisation, and cost-cutting strategies. Managers may minimise stockouts, assure product availability, and optimise inventory levels while taking capacity constraints and production costs into account.It offers information on how adjustments to production procedures, capacity use, and inventory allocation might result in cost reductions.It assists managers in determining the impact of variable demand, capacity constraints, or cost variations on inventory management. Organizations may use this information to establish risk-mitigation measures such as safety stock management, contingency planning, and alternate sourcing choices.The model serves as a framework for assessing and improving supply chain performance. Managers may identify areas for improvement and apply specific initiatives to increase overall performance by analysing key performance metrics such as inventory turnover, order fulfilment rates, and manufacturing cost ratios

Ultimately, it helps businesses to make efficient decisions about inventory management, resource allocation, and cost reduction. Managers may boost efficiency, cut costs, enhance customer happiness, and establish a competitive edge in their respective sectors by recognising the interdependencies between storage space, production expenditures, and other restrictions.

One of the possible extensions of this chapter is to establish particular conditions that guarantee the global optimality of solutions. It is also conceivable to consider the life cycle to be a stochastic function. When this model would cover the time value of money and inflation, a better reflection of real-life scenarios could be presented.

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