A HYBRID ITERATIVE ALGORITHM FOR RECONSTRUCTION OF X-RAY COMPUTED TOMOGRAPHY

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ABSTRACT

Image reconstruction is an important part of computed tomography imaging systems, which converts the measured data into images. Because of high computational cost and slow convergence of iterative reconstruction algorithms, these methods are not widely used in practice. In this paper, we propose a hybrid iterative algorithm by combining multigrid method,Tikhonov regularization and Simultaneous Iterative Reconstruction Technique (SIRT) for reconstruction of the computed tomography image that reduces this drawback. To do so, we reduce the time and the volume of computations considerably by finding astable and appropriate starting point.The experimental results indicate that the proposed iterative algorithm has more rapid convergence and reconstructs high quality images in short computational time than the classical ones.

Keywords: Computed Tomography, iterative methods, reconstruction.

1.0 INTRODUCTION

Computed tomography (CT) is a mathematical technique that combines a series of projections of the body taken from many different angles to produce cross-sectional images of the internal structure of a body. A projection is the line integral of an object in a given direction [1,2]. In CT imaging systems, X-rays pass through the patient's body. The body absorbs some amount of X-ray energy and the remainder, after passing through the body,is measured by the detector. After being digitized, it is stored in the computer memory as a signal. This isdone for different angles until the information of the given areas is completely collected [3,4]. After data acquisition process, the series of projections are then used as an input for reconstruction methods. Reconstruction methods are important parts of any computed tomography (CT) system, which converts the measured data into images (see Fig. 1) [5].

Generally, reconstruction methods for CT images can be classified as analytical or iterative. The difference between two classes is in accuracy and computational time. Analytical methods such as filtered back-projection (FBP) have low computational cost. They can reconstruct images with high quality when the number of projections is large, projections are uniformly distributed over 180° or 360° and noise level is low.Otherwise, reconstructed images have artifacts, noiseand low quality [6,7,8].

Iterative methods are more suitable for the reconstruction of CT images when data is noisy, there are low numbers of projection or they are not uniformly distributed over 180° or 360°. But, according to the high computational cost and slow convergence of iterative methods, these are not widely used in practice [6,7,9].Iterative methods aredividedinto two categories:Algebraic Reconstruction Techniques (ART) andSimultaneous Iterative Reconstruction Techniques (SIRT). In ART, the unknown vector (image) is updated based on one equation in each step and the result is used in the subsequent calculations.In SIRT, information from all equations is used in each step for updating the unknown vector [10,11]. W. Lei and et al. proposed hybrid reconstruction algorithm by combining Tikhonov regularization theory and SIRT to improve the quality of Electrical Capacitance Tomography reconstructed image (called as TSIRT here) [12]. W.Guoand et al. proposed an improved version of SIRT for improving CT image reconstruction [13].

Acceleration of iterative reconstruction methods is an active area of research. The goal of this paper is to accelerate the convergence speed of iterative reconstruction techniques, and to achieve better-reconstructed image quality. To do so, we propose a hybrid iterative reconstruction algorithm by combining multigrid method,Tikhonov regularization and SIRT. In addition,we use some concepts in digital imaging and interpolation.

The rest of the paper is organized as follows: In section 2, the formulation of problem is described. Section 3 discussesthe proposed method, Section 4 is about simulations and experimental results and finally section 5 is comprised of some conclusions.

2.0 FORMULATION OF PROBLEM

Tomographic imaging reconstructs a function from its projections collectedfrom different angles. This approach was first introduced by Radon in 1917. When an X-ray beam travels through tissues, the attenuation of its intensity can be mathematically represented by

$$p_i = I_0\, exp\left(-\int_L f(x,y)dl\right) \tag{1}$$

where$I_0$is the intensity of the incident X-ray beam, $p_i$is the beam intensity at the detector, $f\,(x,y)$ is the distribution of the X-ray attenuation coefficient inside the body, and $L$is the straight line through which beam travels. The reconstruction problem is to determine the values of the function $f(x,y)$ from the set of the projection data p (see Fig. 1)[8,14].

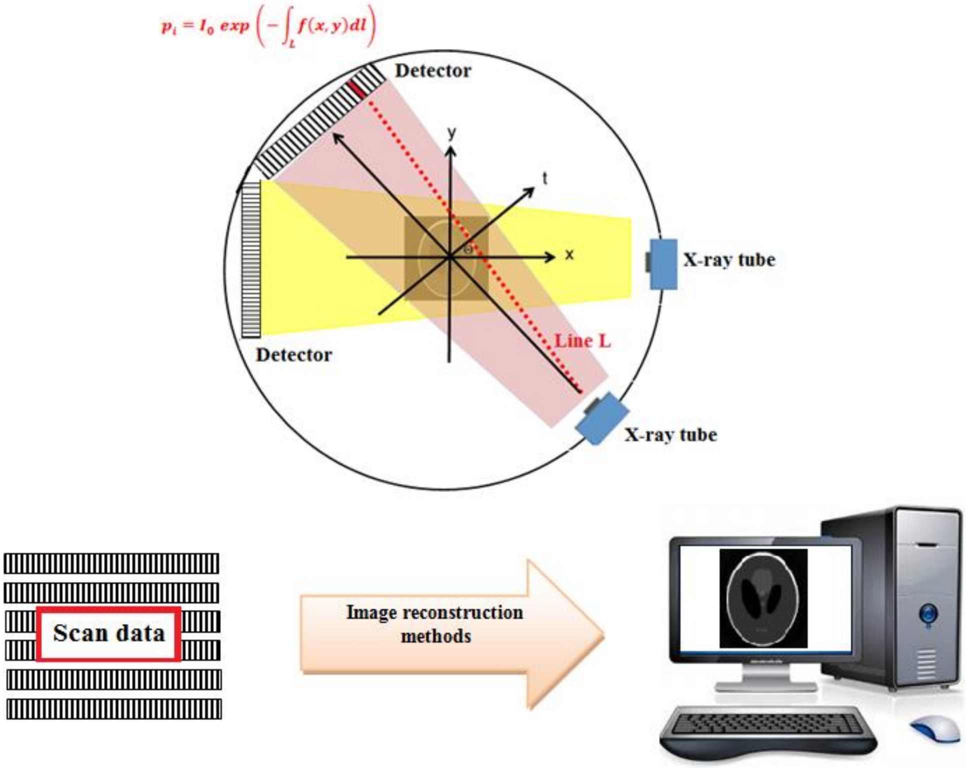


Fig. 1:Reconstruction of computed tomography

In algebraic modeling of tomography, a function $f(x,y)$ is discretized into an$N \times N$Cartesiangrid of pixels. Suppose $f = [f_1, f_2, \dots, f_n]^T$ is the unknown image to be reconstructed, $p = [p_1, p_2, \dots, p_m]^T$ is measured date or projection data, $A = \left[a_{ij}\right]_{m \times n}$ is the coefficient matrix created from modeling of ray geometry (the intersection of the i^{th}ray with thej^{th}pixel). $n = N \times N$and$m$are the number of pixels in image f and the total number of rays, respectively. The weighting coefficients a_{ij} have value [0,1] and represent the contribution of the j^{th} cell to the i^{th} ray integral. So, the reconstruction problem can be expressed in the following linear system of equations[3,4,15]:

$$A_{m \times n} f_{n \times 1} = p_{m \times 1} \tag{2}$$

Because $m < n$, the system $Af = p$ is under-determined; we consider the Linear Least Squares problem to find the value of$min_f \|Af - p\|_2$. The matrix A is the discretization of the Radon transform, so it is ill-posed. Since a small number of pixels contributes in any ray, most a_{ij}'s are zero and matrix A is sparse. To solve a sparse, large and ill-posed system of equations, iterative methods must usually be employed.

2.1 Simultaneous Iterative Reconstruction Technique (SIRT)

In Simultaneous Iterative Reconstruction Techniques (SIRT), information from all equations isused simultaneously in each step to updateimage$f$. This method has the following scheme:

$$\forall j \quad f_j^{(k+1)} = f_j^{(k)} + \lambda \frac{1}{m} \sum_{i=1}^{m} \frac{p_i - \sum_{l=1}^{n} a_{il} f_l^{(k)}}{\sum_{l=1}^{n} a_{il}^2} a_{ij} \tag{3}$$

where $f_j^{(k+1)}$is the current estimation of the attenuation coefficient associated with the j^{th} projection ray in iteration $k+1$, a_{ij} is the weighting factor representing the contribution of the j^{th} cell to the i^{th} ray integral, and λ is the relaxation parameter that controls the convergence rate [16,17].

W. Guo and et al. improved SIRT [13]. From an algebraic view, the denominator in Eq. 3 is associated with the matrix A. Since the matrix A is large, sparse and ill-posed, any little error in the matrix elements may result in the deviation from its exact solution and convergence to a naive solution. Therefore, to consider the compensation of obtained f_j in the last iteration; the denominator of Eq. 3is adjusted to:

$$\forall j \quad f_j^{(k+1)} = f_j^{(k)} + \lambda_k \frac{1}{m} \sum_{i=1}^{m} \frac{p_i - \sum_{l=1}^{n} a_{ij} f_l^{(k)}}{\sum_{l=1}^{n} a_{il} \frac{f_l^{(k)}}{f_j^{(k)}}} a_{ij} \tag{4}$$

2.2 Tikhonov regularization

The most commonly used method to solve discrete ill-posed problems with datapolluted by noiseis Tikhonov regularization.The Tikhonov regularization method converts the ill-posed system of $Af = p$ to the following minimization problem:

$$min_f \{\|Af - p\|_2^2 + \alpha^2 \|f\|_2^2\} \tag{5}$$

where$\alpha \geq 0$ is the regularization parameter andvalueα^2keepsa balance between$\|Af - p\|_2^2$and $\|f\|_2^2$. The regularized solution of Eq. 5 is given as

$$f = (A^T A + \alpha^2 I)^{-1} A^T p \tag{6}$$

where$I$ is the identity matrix [18].

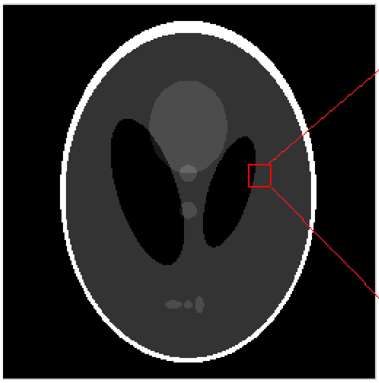
3.0 PROPOSED METHOD

Fundamentally, the reconstruction problem of CT images is aninverseproblem and leads to solving a system of linear equations$Af = p$ by iterative methods. The nature of the system has a significant impact on the quality and speed of the reconstruction process. Unfortunately, this system is large and sparse and, due to noise in the projections,ill-posed.On the other hand,iterative methods to solve this system of equations, such as SIRT, are computationally expensive and have a slow convergence to the solution.

The goal of this paper is to accelerate the convergence speed of iterative reconstruction techniques, and to achieve better reconstructed image quality.To do so, we propose a hybrid iterative reconstruction algorithm whichcombines the multigrid method,Tikhonov regularization and SIRT. First by using some concepts in digital imaging, interpolation and multigrid method, we reduce unknowns (image f) and thus, the linear system can be solvedin a more rapid time. Because the system is ill-posed, Tikhonov regularization theory is used to solve it. Finally, SIRT is used to accelerate reconstruction and improve reconstructed image quality.The proposed method is called MTSIRT.

The hybrid proposed algorithm consists of two stages. In the first stage, we construct a system of linear equations in smaller dimension and solve it. In the second stage, we use the solution resulting from the first step as a starting point for the original system of linear equations.

As computed tomography images are grayscale images, the intensity values of pixels are real numbers between 0and 1. Usually, pixels in adjacent neighborhoods (e.g. in blocks of 2×2, 4×4 or 8×8) have intensity values very close together (see Fig. 2) [15]. In our proposediterative method, we use this property to reduce unknowns and thus the linear system can be solved faster. In addition, we find an appropriate starting point for the iterative reconstruction method.



0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.20
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.20
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.20	0.20
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.20	0.20
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.20	0.20	0.20
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.20	0.20	0.20	0.20
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.20	0.20	0.20	0.20
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.20	0.20	0.20	0.20	0.20	0.20
0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20

Fig. 2: Examples of intensity values of 2×2, 4×4 and 8×8blocks.

As shown in Fig. 3(a), the object is discretized into an$N \times N$Cartesian grid of pixels($N \times N$ is the resolution to be reconstructed). Let $f_1^h,\ f_2^h, \dots, f_n^h$ be the pixels on the fine grid Ω^h where $n = N \times N$. The i^{th} ray passes through a slice of the object. The object absorbs some amount of ray energy and the remainder passes through the object and is measured by the detector. For these settings, we construct a system of linear equations
$A^h f^h = p^h$ wherep_i^h is the line integral along the i^{th}rayand the entry a_{ij}^h of A^h is the weighting factor representing the contribution of the j^{th} pixel along the i^{th} ray [19,20].

Suppose $n = 2^t \times 2^t = 2^{2t}$. According to Fig. 3(b),$f_1^H,\ f_2^H, \dots, f_{n/4}^H$ are pixels of the coarse grid Ω^Hobtained by considering the bigger pixels formed by a 2×2 block of adjacent pixels from the fine grid. Now, we should construct coarse grid matrixA^H. The entry a_{ij}^H of matrix $A^H$is defined according to the indices of fine grid pixels from which the coarse grid pixels are built, as follow:

$$a^H_{i,2k+j} = a^h_{i,2kN+2j-1} + a^h_{i,2kN+2j} + a^h_{i,(2k+1)N+2j-1} + a^h_{i,(2k+1)N+2j} \qquad (7)$$
$$j = 1, \dots, \frac{n}{4}, \qquad k = 0, \dots, (\frac{n}{4} - 1)$$

Let P, R and$m = P \times R$ be the number of projections, number of rays in each projection and total number of rays, respectively. Now select a subset of the original projection vectorp^h. We use the projectionsonly in angles
$\theta^H_i = 8i\pi/P$, $i = 1, 2, \dots, [P/8]$ from projection angles$\theta^h_i = i\pi/P$, $i = 1, 2, \dots, P$ and use$R/(2 \times 2)$ rays in each selected projection corresponding to exploiting 2×2 blocks. Thus, the new projection vector $p^H$is constructed in this way.

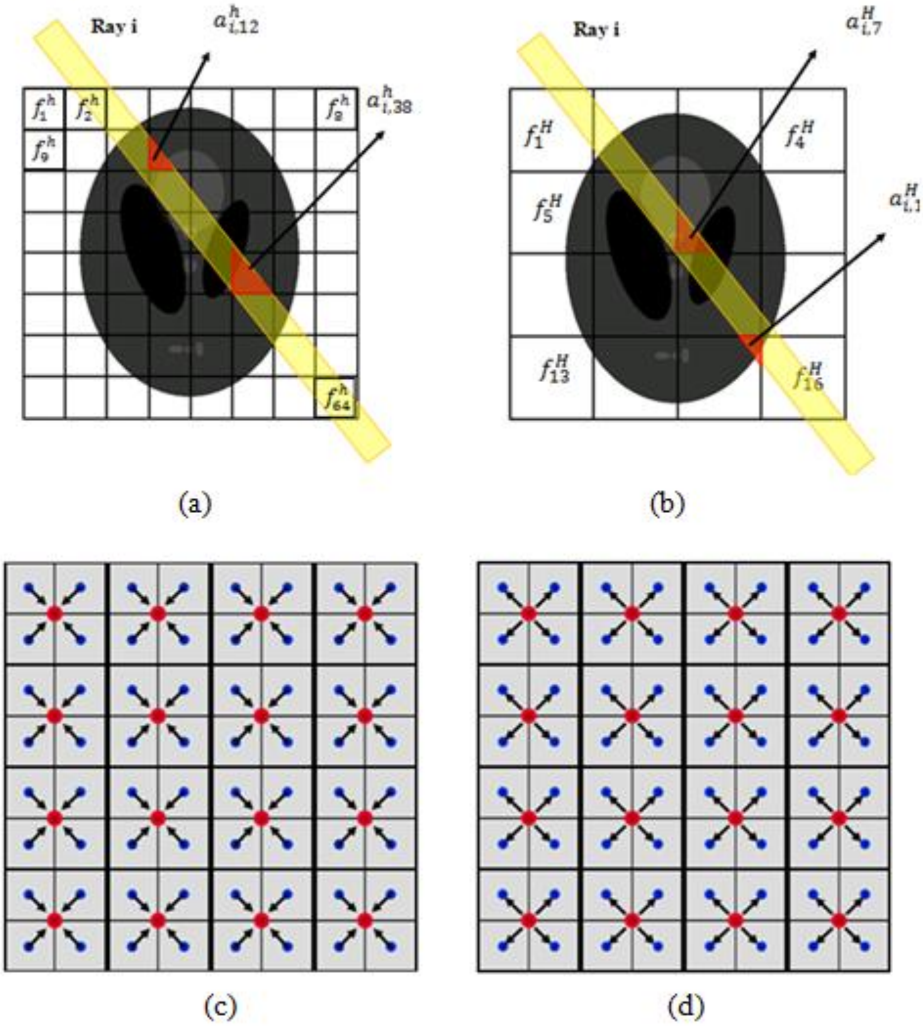


Fig. 3:(a) Fine grid Ω^h, (b) coarse grid Ω^H, (c) the coarse grid points are considered as representative of block of fine grid points, (d) the values of coarse grid points are distributed to blocks of fine grid points.

Using the concepts of multigrid method in the numerical linear algebra, we can reduce unknowns and thus, the system of linear equationscan be solvedmore rapidly. The size of new system of linear equations$A^H f^H = p^H$is 1/128 of the original system of linear equations.

The matrix $A^H$is ill-posed and the projection $p^H$is polluted by noise. This noise can be caused by discretization or measurement errors.Therefore, we use Tikhonov regularization (Eq. 6) to find the solutionf'^H. This solution, used as the starting point for SIRT, accelerates the convergence of SIRT to the optimal solution and improves the quality of reconstructed image.

Suppose $f^H$is the resulting solution of coarse grid Ω^Hin the first stage of the proposed algorithm. Now, the values of coarse grid points $f^H$are distributed to respective blocks of fine grid points (see Fig. 3(c) and (d)). $f^{h,(0)}$is used asastarting pointto solvethe original system of linear equations$A^h f^h = p^h$. Algorithm 1 shows the proposed iterative algorithm.

Algorithm 1 Proposed iterative algorithm: MTSIRT

// Solve large, sparse and ill-posed system of linear equation $A^h_{m\times n} f^h_{n\times 1} = p^h_{m\times 1}$ from reconstruction of CT images

P= *Number of projections*
R= *Number of rays in each projection*
$m = P \times R$ =*total projection*
// *Size of reconstructed image*= $N \times N$
$n=N \times N$
// *Measure projection data and store in projection vector p*
Measure the projection $p^h(t,\theta)$ *for* $\theta_i^h = i\pi/P\,, i = 1,2,\ldots,P$
// *Select projection data* p^H
Select the projection $p^H(t,\theta)$ *for* $\theta_i^H = 8i\pi/P\,, i = 1,2,\ldots,[P/8]$
// Coarse grid matrix A^H
for $i \in \theta_i^H$
  for $j = 1$ *To* $n/4$
    for $k = 0$ *To* $(n/4) - 1$
      $a^H_{i,2k+j} = a^h_{i,2kN+2j-1} + a^h_{i,2kN+2j} + a^h_{i,(2k+1)N+2j-1} + a^h_{i,(2k+1)N+2j}$
    end
  end
end
$f^{H,0} = (1,1,\ldots,1)$ // *Set an initial guess of the image* f^H.
// *Solve new system of linear equation* $A^H f^H = p^H$ *by Tikhonov regularization*
f'^H =*TikhonovRegularization* $(A^H, p^H, f^{H,(0)})$
// *Solve new system of linear equations* $A^H f^H = p^H$ *by SIRT method.*
f^H = *SIRT* (A^H, p^H, f'^H)
//*The coarse grid value* f^H *distributes its value to surrounding fine grid points*
$v = 1$
for $i = 1$ *To* N
  for $j = 1$ *To* N
    $\left\{ f^{h,(0)}_{j+(i-1)N}, f^{h,(0)}_{(j+1)+(i-1)N}, f^{h,(0)}_{j+iN}, f^{h,(0)}_{(j+1)+iN} \right\} \leftarrow f^H_v$
    $v = v + 1$
  end
end
// *Solve new system of linear equations* $A^h f^h = p^h$ *by Tikhonov regularization*
f'^h =*TikhonovRegularization* $(A^h, p^h, f^{h,(0)})$
// *Solve new system of linear equations* $A^h f^h = p^h$ *by SIRT method.*
f = *SIRT* (A^h, p^h, f'^h)
Reconstructed image=Reshap(f, N, N)

Fig. 4: MTSIRT algorithm

4.0 EXPERIMENTAL RESULTS

In this section, our experimental results are presented. We use MATLAB for implementation of reconstructed methods.We use the Shepp-Logan phantom and Chest phantom as benchmarks to evaluate the quality of images reconstructed. Fig. 5(a) showsthe Shepp-Logan phantom, a famous model of the brain in medical tomography.The size of phantomsis 256×256 pixels.64 projections within 180° with 2.8125° interval and 367 parallel rays per projection were taken for the reconstruction process. The size of matrix A is 23488×65536 (see Fig. 5(b)).To evaluate speed, accuracy and the quality of the reconstruction methods, we use three criteria: computational time, Mean Square Error (MSE) and Correlation Coefficient (CC).

- The computational time: One of thecriteria to compare severalalgorithmstosolveaproblem is the computational time. An Intel® Core™ i7-870 Processor (8M Cache, 2.93GHz) and 8GB of RAM are used in our experiments.

- Mean Square Error (MSE): One common measure of the quality and accuracy of a reconstruction technique is MSE of the reconstructed image related to the original image.MSE is calculated by the following formula:

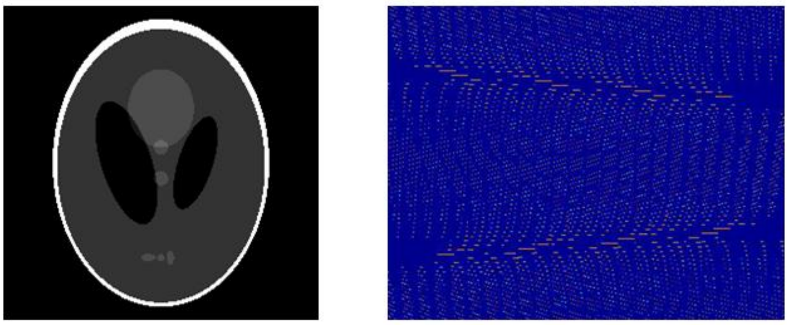
$$MSE = \frac{\sqrt{\sum_i \sum_j (O_{ij} - f_{ij})^2}}{N^2} \tag{8}$$

  whereO_{ij} and f_{ij} are theintensity of $(i,j)^{th}$ pixel of the original and reconstructed image respectivelyand N^2 is number of pixels in image.

- Correlation Coefficient (CC): Correlation Coefficient has been widely used in image processing to compare thesimilarity between two images. We are going to answer the following questions: If there exists a linear relationship between two images, what can we say about the strength and the direction of this relationship?The sample correlation coefficient is calculated by the following formula:

$$CC = \frac{\sum_i \sum_j (O_{ij} - \bar{O})(f_{ij}^k - \overline{f^k})}{\sqrt{\sum_i \sum_j (O_{ij} - \bar{O})^2 (f_{ij}^k - \overline{f^k})^2}} \tag{9}$$

  wheref_{ij}^k and $\overline{f^k}$are theintensity of $(i,j)^{th}$ pixel and meanintensity of reconstructed image in iteration k ,and O_{ij} and$\bar{O}$are intensity of the $(i,j)^{th}$ pixel and meanintensity of the original image. The correlation coefficient varies between -1 and 1, with magnitude indicating the strength of linear relationship and sign indicating the direction. $CC = 1$if two images are absolutely identical, and $CC = 0$ if they are completely uncorrelated, and $CC = -1$ if they are completely anti-correlated [19].



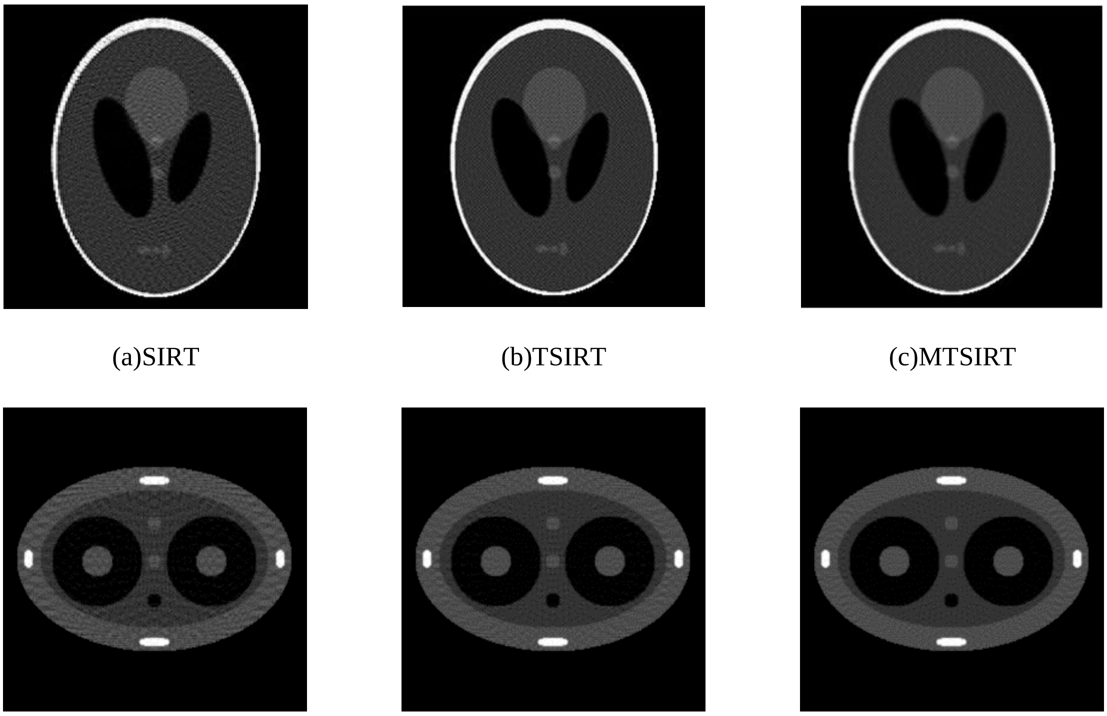
(a) (b)

Fig. 5: (a) Shepp-Logan phantom, (b) structure of matrix A.

Table 1shows the computational time, MSEand CC of SIRT, TSIRT and MTSIRT methods for Shepp-Logan and Chest phantom.The Time, MSE and CC in Table 1indicate values of computational time, Mean Square Error and Correlation Coefficients in the stopping time of algorithms based on the stopping criteria. Fig. 6shows the reconstructed images for bySIRT, TSIRT and MTSIRT methods.

Table 1. The iterations, time, Mean Square Error and Correlation Coefficients form reconstruction methods

Phantom	Algorithm	Iterations	Time (Seconds)	MSE	CC
Shepp-Logan	SIRT	184	17.8741	0.0332	0.9267
	TSIRT	141	9.5023	0.0243	0.9503
	MTSIRT	91	5.6388	0.0232	0.9632
Chest	SIRT	201	20.4349	0.0330	0.9337
	TSIRT	153	13.1552	0.0242	0.9553
	MTSIRT	98	6.8740	0.0227	0.9681



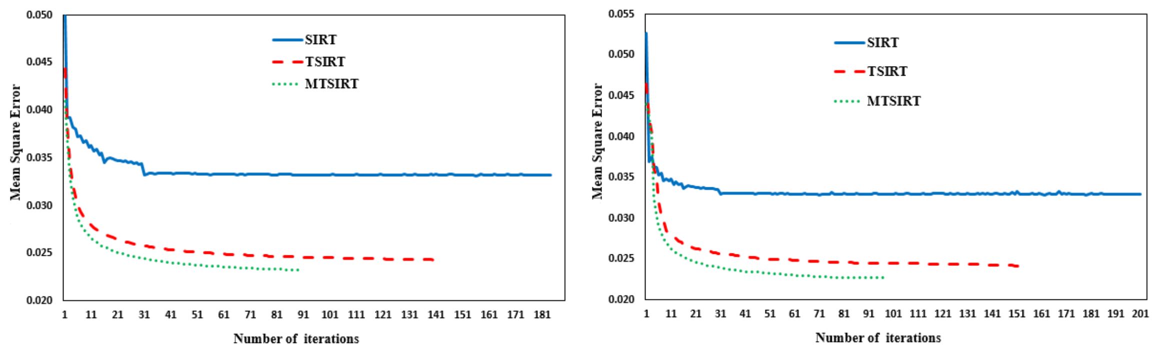
(a)SIRT (b)TSIRT (c)MTSIRT

(d)SIRT (e)TSIRT (f)MTSIRT

Fig. 6: Reconstruction of 256×256 Shepp-Logan and Chest phantomsby SIRT,TSIRT and MTSIRTmethods

The convergence of the iterative reconstruction method is considered in terms of Mean Square Error and visual image quality. The convergence speeds of the SIRT, TSIRT and MTSIRT methods are shown in Fig. 7and Fig. 8in terms of the number of iterations.

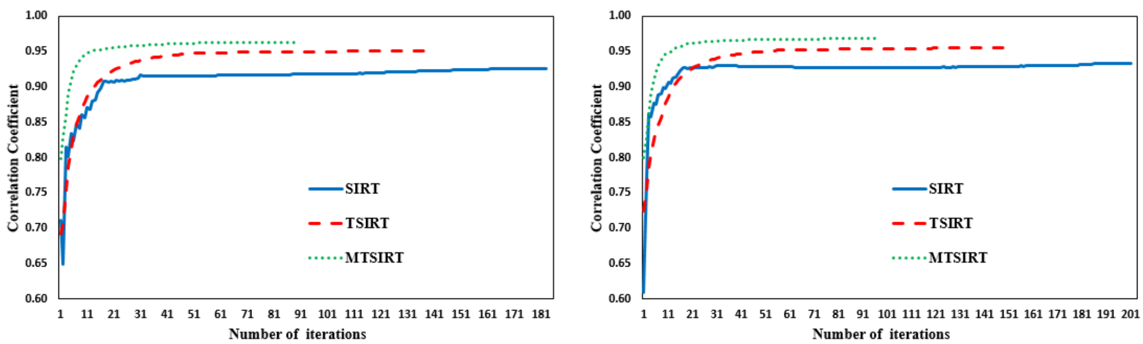
Fig. 7 shows the Mean Square Error plots of reconstructed image with SIRT, TSIRT and MTSIRT methods. In iterative methods, determining the optimal number of iterations has a significant impact on finding a meaningful answer. On the other hand, according to the concept of semi-convergence, the number of iterations is the regularization parameter for the iterative methods [17]. The iteration column in Table 1 shows the optimal iteration. The results indicate that SIRT and TSIRT methods are to converge far much slower than the MTSIRT.



(a)Shepp-Logan phantom (b)Chest phantom

Fig. 7: The Mean Square Errorplots of reconstructed images

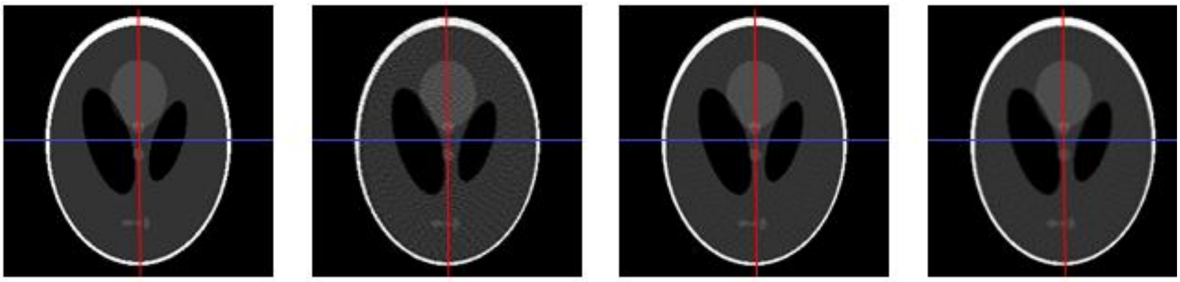
Fig .8shows the Correlation Coefficient plots of reconstructed images by SIRT, TSIRT and MTSIRT methods. The value of CC increases monotonically towards 1.0 as the number of iterations increases. Whatever the value of CC is closer to 1, the similarity between reconstructed image andShepp-Logan and Chest phantomsisincreased, which concludes better quality of reconstructed image.



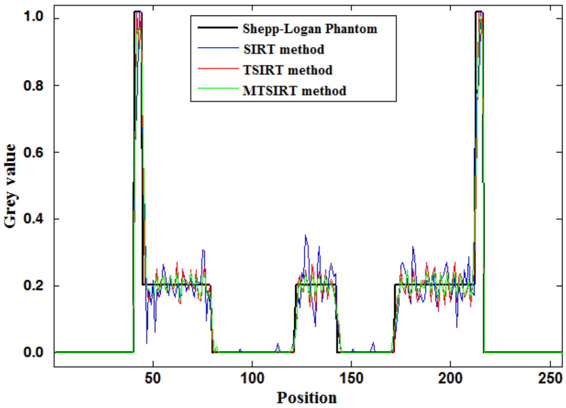
(a) Shepp-Loganphantom (b) Chest phantom

Fig. 8: The Correlation Coefficient plots of reconstructed images

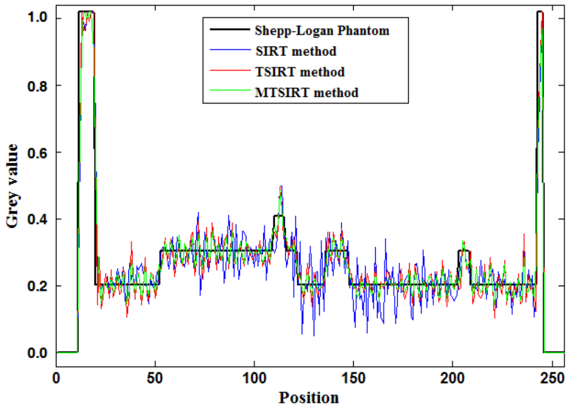
Fig. 9 and Fig. 10 show the profile of the 128th row and column of Shepp-Logan and Chest phantom and reconstructed images by SIRT, TSIRT and MTSIRT methods. This 1D profile plot shows the quality of the reconstructed image. Regarding tothese figures, it is evident thatthe profile of MTSIRT matches well to the profile of original phantoms. Therefore, the reconstructed image quality by the MTSIRT method is more efficient than the SIRT and TSIRT method.



(a) Shepp-Logan phantom (b) SIRT (c) TSIRT (d) MTSIRT

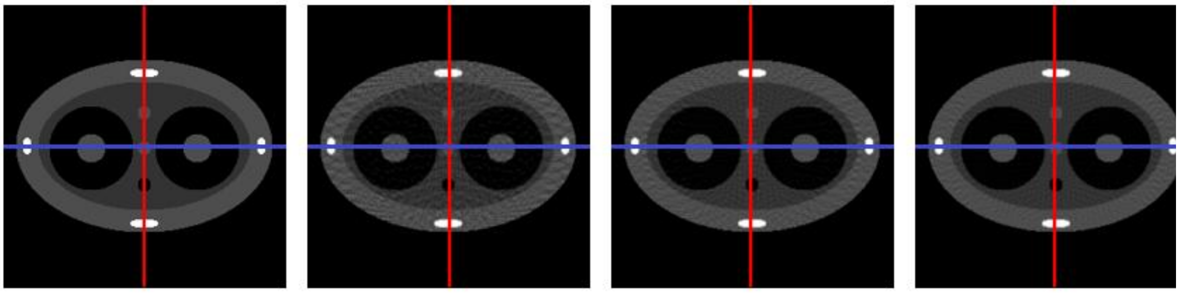


(e) Profile of 128th row (blue lines)

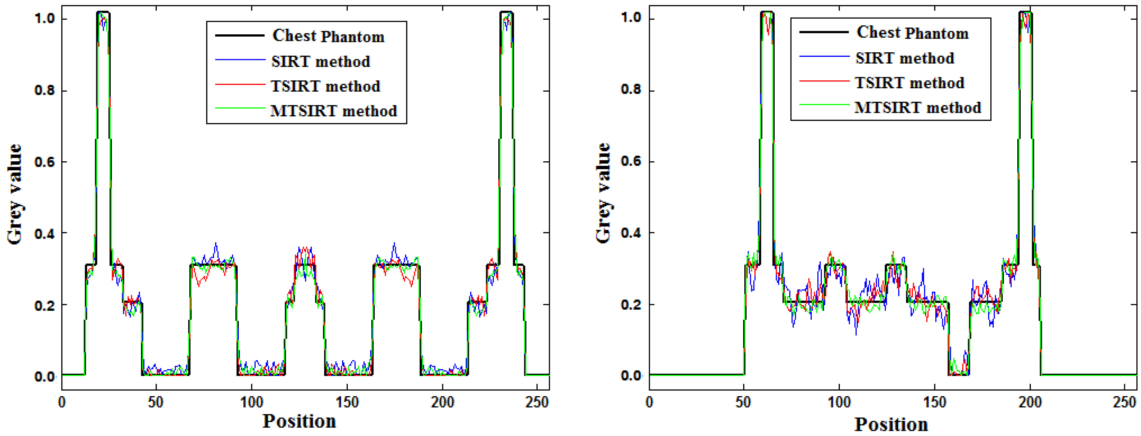


(f) Profile of 128thcolumn (red lines)

Fig. 9: Pixel-intensity profiles of 128throw and column ofShepp-Logan phantom and reconstructed images by SIRT, TSIRT and MTSIRT methods



(a) Chest phantom (b) SIRT (c) TSIRT (d) MTSIRT



(e) Profile of 128th row (blue lines) (f) Profile of 128thcolumn (red lines)

Fig. 10: Pixel-intensity profiles of 128th row and column of Chest phantom and reconstructed images by SIRT, TSIRT and MTSIRT methods

According to experimental results, it is obvious that the MTSIRT method consumes less computational time compared to SIRT and TSIRT methods, and in terms of quality, is better. Therefore, by finding an appropriate starting point using concepts in digital imaging, interpolation and multigrid methodandusingTikhonov regularization to overcome ill-posedness, we could reduce the time and volume of computations considerably, and accelerate the convergence speed of SIRT, which is one of the main challenges in iterative reconstruction methods.

5.0 CONCLUSIONS

In this paper, we proposed ahybrid iterative algorithm (MTSIRT) by combining multigrid method,Tikhonov regularization and SIRT methodsfor reconstruction problem of computed tomography image. The drawbacks of iterative reconstruction algorithms are high computational cost and slow convergence. To reduce these drawbacks, we try to find appropriate starting points, whichreduce the time and the volume of computations considerably, accelerate convergence of iterative methods, and achieve a better quality of image reconstruction. To do so, we usedconcepts in digital imaging, interpolation and multigrid method to reduce unknowns. We obtaineda coarse grid model from the algebraic modeling of tomography and then distributedits value to surrounding fine grid points and finally, the reconstructed image is regarded as an intermediate result and is used for reconstruction high-resolution images. As the linear system of equationsis ill-posed, Tikhonov regularization was used to find a stable starting point for SIRT. Based on experimental results, the proposed iterative method reconstructs CT images with quicker convergence and higher quality images in shorter computational time than the classical ones. Future work includes faster implementation using parallel computing and GPU.

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